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UNSTEADY-STATE HEATING OF A FINITE DISC BY CONTINUOUS AND REPETITIVELY PULSED LASER RADIATION

William T. Laughlin
Capt USAF

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TECHNICAL REPORT NO. AFWL-TR-72-236

March 1973

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
New Mexico

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FOREWORD

This research was performed under Program Element 63605F, Project 317J, Task 10.

The inclusive dates of research were September 1971 to November 1972. This report was submitted 1 December 1972 by the Air Force Weapons Laboratory Project Officer, Captain William T. Laughlin (LRE).

This technical report has been reviewed and is approved.

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ABSTRACT

(Distribution Limitation Statement B)

The heating of a solid body by a laser beam has been analyzed using a generalized two-dimensional solution of the heat conduction equation for a finite right circular cylinder. This series-eigenvalue solution includes time-dependent flux boundary conditions and reradiation from all surfaces. Temperature solutions are derived for three spacial laser beam profiles applied to continuous wave, pulsed, and repetitively pulsed laser heating. Computer programs developed to evaluate these temperature solutions are presented.

CONTENTS

SECTION I

INTRODUCTION

The investigation of the effects of continuous, pulsed, and repetitively pulsed lasers on materials has brought about a renewed effort to solve a particular class of heat flow problems. These problems vary greatly in complexity depending on the thermal properties of the material of interest; the temporal history of the laser flux; the spacial intensity profile of the laser beam; and the shape, size, and optical properties of the body being irradiated. Few laser heating problems can be considered steady state, and most are two- or three-dimensional in nature.

Ready (Ref. 1) has compiled an excellent summary of heat transfer solutions applicable primarily to pulsed laser heating problems. There are solutions for two-dimensional radially symmetric heating by circular and Gaussian beam profiles as well as one-dimensional cases. Most of these have been solved for the semi-infinite solid case because of the emphasis on pulsed laser heating, although some solutions are presented for the finite thickness plate, including surface losses from reradiation or convection. To obtain these concise analytical solutions, it was assumed that the heat flow obeys the classical heat transfer equations with no phase changes occurring. Material thermal properties are independent of temperature. Despite further assumptions about the temporal shape of the laser pulse and small absorption depths at surface (excellent assumption for metals), some of the solutions presented still require the numerical evaluation of integrals.

Carslaw and Jaeger (Ref. 2) give a one-dimensional transient heat transfer solution for a finite thickness plate with a prescribed flux entering one surface and no heat flow across the other surface. Again, constant thermal properties were assumed, and no phase changes are permitted. If absorption lengths for the laser energy are assumed to be short, this solution can be used to model continuous wave laser heating with the condition that the one-dimensional criteria are met. This requires that for a material of given thermal properties, the average laser power be sufficiently high and the sample be sufficiently thin (Ref. 3).

An additional one-dimensional heat transfer solution for a finite thickness plate applicable to pulsed laser heating can be derived by (1) using the above prescribed flux solution for the duration of the pulse, and (2) applying the resulting temperature profile at the end of the pulse as an initial condition to a finite thickness plate solution (Ref. 2) for any initial temperature profile and both surfaces insulated. This second solution then describes the diffusion away from the front surface of the heat deposited by the pulse. Again the same criteria for one-dimensionality must be obeyed.

Since in most laser experiments the one-dimensional requirements are not met, it is desirable to have a full transient two-dimensional heat transfer solution to theoretically describe laser heating. N. Y. Ölcer (Refs. 4,5) has derived very general solutions of the appropriate heat conduction equation for the heating of a finite solid body of arbitrary shape, any initial temperature profile, and time dependent surface conditions. In this work he has specialized the solution to that for a finite right circular cylinder that retains the very general time dependent boundary conditions which permit his results to be applied with appropriate simplification to the laser heating problem.

It is the intent of this report to show how the work of Ölcer can be simplified and yet be made to model laser heating in as general a way as possible. Specific examples are then derived for heating with three different spacial laser beam intensity profiles and for continuous wave, pulsed, and repetitively pulsed lasers. The computer programs used to evaluate the solutions are described, and two examples are given in the appendix.

Finally, the applications section outlines ways to use the existing programmed solutions. It should be remembered that many other solutions for laser heating problems can be derived from Ölcer's basic solution in addition to the ones presented here.

SECTION II

APPLICATION OF ÖLCER'S GENERAL SOLUTION TO THE LASER HEATING PROBLEM

1. SIMPLIFICATION OF ÖLCER'S SOLUTION

The heat conduction equation

$$\nabla'^2 T'(\vec{r}',t') + Q'(\vec{r}',t') = \frac{1}{\kappa}\frac{\partial T'(\vec{r}',t')}{\partial t'} \qquad \begin{array}{l}\vec{r}' \text{ in } R' \\ t' > 0\end{array} \qquad (1)$$

for the heating of a finite isotropic, homogeneous solid body, R', of arbitrary shape with constant thermal properties has been solved by N. Y. Ölcer (Refs. 4, 5). The following general boundary conditions apply on the surfaces $S_i'(i=1,2,\ldots,q)$ of the body R',

$$\left(K\frac{\partial}{\partial n_i'} + h_i'(\vec{r}')\right)T'(\vec{r}_i',t') = f_i'(\vec{r}',t') \qquad \begin{array}{l}t' > 0 \\ \vec{r}' \text{ on } S_i'(i=1,2,\ldots,q)\end{array} \qquad (2)$$

with an initial temperature distribution

$$T'(\vec{r}',t') = F'(\vec{r}') \qquad \vec{r}' \text{ in } R' \text{ and on } S_i';\ t' = 0 \qquad (3)$$

The terms of equations (1), (2), and (3) are defined as follows:

$T'(r',t')$ = unsteady temperature distribution in the region R'

∇'^2 = the Laplacian in $\vec{r}'$-space

$Q'(\vec{r}',t')$ = heat production per unit time per unit volume in R'

K = thermal conductivity

κ = thermal diffusivity

$\vec{r}'$ = position of a point in the region R'

t' = time

q = number of surfaces S_i' bounding R'

$f_i'(\vec{r}',t')$ = heat entering surfaces S_i' per unit time per unit area

$h_i'(\vec{r}\,')$ = a coefficient of heat transfer (radiation and convection) from surfaces S_i'

n_i' = direction of the outward normal to surface S_i

$F'(\vec{r}\,')$ = prescribed initial temperature distribution

For the convenience of removing constants from equations (1) and (2) and simplifying its solution, the following nondimensional parameters are defined:

$$f_i(\vec{r},t) = a\, f_i'(\vec{r}\,',t')/T_0K$$

$$h_i(\vec{r}) = a\, h_i'(\vec{r}\,')/K$$

$$Q(\vec{r},t) = a^2 Q'(\vec{r}\,',t')/T_0K$$

$$\vec{r} = \vec{r}\,'/a$$

$$t = t'\kappa/a^2$$

$$T(\vec{r},t) = T'(\vec{r}\,',t')/T_0$$

$$F(\vec{r}) = F'(\vec{r}\,')/T_0 \qquad (4)$$

where a is a characteristic dimension of the body R' and T_0 is some reference temperature. Equations (1), (2), and (3) can now be written in dimensionless form.

$$\nabla^2 T(\vec{r},t) + Q(\vec{r},t) = \frac{\partial T(\vec{r},t)}{\partial t} \qquad \begin{matrix} \vec{r} \text{ in } R \\ t > 0 \end{matrix} \qquad (5)$$

$$\left(\frac{\partial}{\partial n_i} + h_i(\vec{r})\right) T(\vec{r},t) = f_i(\vec{r},t) \qquad \begin{matrix} \vec{r} \text{ on } S_i (i=1,2,\ldots,q) \\ t > 0 \end{matrix} \qquad (6)$$

$$T(\vec{r},t) = F(\vec{r}) \qquad \begin{matrix} \vec{r} \text{ in } R \text{ and on } S_i \\ t = 0 \end{matrix} \qquad (7)$$

where R, S_i, n_i, and ∇^2 in nondimensional $\vec{r}$-space correspond to the same quantities or operations primed in $\vec{r}'$-space with dimensions. All further discussion of solutions of the heat transfer equation in this report will be in terms of dimensionless, unprimed quantities. In the numerical evaluation of a solution equation (4) must be used first to translate input parameters to dimensionless form, and second to return the calculated temperature field to actual temperatures.

The solution to equations (5), (6), and (7) is from reference 5.

$$T(\vec{r},t) = \sum_{m}^{\infty} A_m \psi_m(\vec{r}) \exp(-\mu_m^2 t) \left\{ \iiint_{\vec{R}} \psi_m(\vec{r}) F(\vec{r}) d\vec{R} \right.$$

$$\left. + \int_0^t \exp(\mu_m^2 \tau) \left(\iiint_{\vec{R}} \psi_m(\vec{r}) Q(\vec{r},\tau) d\vec{R} + \sum_{i=1}^{q} \iint_{S_i} \psi_m(\vec{r}) f_i(\vec{r},\tau) dS_i \right) d\tau \right\} \tag{8}$$

where the eigenfunctions $\psi_m(\vec{r})$ and the non-negative eigenvalues μ_m are obtained by solving the eigenvalue problem

$$(\nabla^2 + \mu_m{}^2)\, \psi_m(\vec{r}) = 0 \qquad \vec{r} \text{ in } \vec{R}$$

$$\left(\frac{\partial}{\partial n_i} + h_i(\vec{r}) \right) \psi_m(r) = 0 \qquad \vec{r} \text{ on } S_i (i=1,2,\ldots,q) \tag{9}$$

and

$$\frac{1}{A_m} = \iiint_{\vec{R}} \psi_m^2(\vec{r}) d\vec{R} \tag{10}$$

This solution has been specialized to the case of a right circular cylinder of radius, a, and thickness, 2b, in cylindrical coordinates (Ref. 5). For the application of this solution to the heating of a disc by a laser, it will be assumed that the spacial intensity profile of the laser beam is symmetrical

about its axis, and therefore, the temperature profile in the heated body also has cylindrical symmetry. Dropping the angular dependence the nondimensional coordinates required are

$$r = r'/a; \qquad z = z'/a$$

and

$$\eta = b/a$$

The heat flow equation to be solved and the necessary boundary and initial conditions corresponding to equations (5), (6), and (7) are as follows.

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} - \frac{\partial}{\partial t}\right)T(r,z,t) + Q(r,z,t) = 0$$

$$0 \leq r < 1 \qquad |z| < \eta \qquad t > 0 \qquad (11)$$

$$\left(-\frac{\partial}{\partial z} + h_1\right) T(r,z,t) = f_1(r,t)$$

$$0 \leq r < 1 \qquad z = -\eta \qquad t > 0$$

$$\left(\frac{\partial}{\partial z} + h_2\right) T(r,z,t) = f_2(r,t)$$

$$0 \leq r < 1 \qquad z = \eta \qquad t > 0$$

$$\left(\frac{\partial}{\partial z} + h_3\right) T(r,z,t) = f_3(z,t)$$

$$r = 1 \qquad |z| < \eta \qquad t > 0 \qquad (12)$$

$$T(r,z,t) = F(r,z)$$

$$0 \leq r \leq 1 \qquad |z| \leq \eta \qquad t = 0 \qquad (13)$$

The solution of equations (11), (12), and (13) for the axisymmetric right circular cylinder which corresponds to equation (8) is

$$T(r,z,t) = \sum_{k}^{\infty} \sum_{n}^{\infty} C_n L_k \phi_n(r) \chi_k(z) \exp\left[-\left(\lambda_n^2+\nu_k^2\right)t\right]$$

$$\cdot \left\{ \int_0^1 \int_{-\eta}^{\eta} \phi_n(r)\chi_k(z)F(r,z)rdrdz + \int_0^t \exp\left[\left(\lambda_n^2+\nu_k^2\right)\tau\right] \right.$$

$$\cdot \left[\int_0^1 \int_{-\eta}^{\eta} \phi_n(r)\chi_k(z)Q(r,z,\tau)rdrdz + \int_0^1 \phi_n(r)\left(f_1(r,\tau) + \chi_k(\eta)f_2(r,\tau)\right)rdr \right.$$

$$\left. \left. + \int_{-\eta}^{\eta} \chi_k(z)f_3(z,\tau)dz \right] d\tau \right\} \tag{14}$$

and the eigenvalue problem of equation (9) becomes

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} + \mu_{kn}^2\right)\psi_{kn}(r,z) = 0 \qquad 0 \le r < 1 \qquad |z| < \eta$$

$$\left(-\frac{\partial}{\partial z} + h_1\right)\psi_{kn}(r,z) = 0 \qquad 0 \le r < 1 \qquad z = -\eta$$

$$\left(\frac{\partial}{\partial z} + h_2\right)\psi_{kn}(r,z) = 0 \qquad 0 \le r < 1 \qquad z = \eta$$

$$\left(\frac{\partial}{\partial r} + h_3\right)\psi_{kn}(r,z) = 0 \qquad r = 1 \qquad |z| < \eta \tag{15}$$

The eigenfunctions $\psi_{kn}(r,z)$ and the eigenvalues μ_{kn} are double index sets where $k,n = 0,1,2,\ldots\infty$. Where h_1,h_2,h_3 are constants, the solution to equation (15) is a product of two eigenfunctions

$$\psi_{k,n}(r,z) = \phi_n(r)\chi_k(z) \tag{16}$$

where the eigenfunction $\phi_n(r)$ is

$$\phi_n(r) = J_0\left(\lambda_n r\right)/J_0\left(\lambda_n\right) \tag{17}$$

and λ_n is the nth non-negative root of

$$h_3 J_0\left(\lambda_n\right) = \lambda_n J_1\left(\lambda_n\right) \tag{18}$$

The eigenfunction $\chi_k(z)$ is

$$\chi_k(z) = \cos\left[\nu_k(\eta+z)\right] + \frac{h_1}{\nu_k}\sin\left[\nu_k(\eta+z)\right] \tag{19}$$

and ν_k is the kth non-negative root of

$$\left(h_1+h_2\right)\nu_k \cos 2\eta\nu_k = \left(\nu_k^2-h_1h_2\right)\sin 2\eta\nu_k \tag{20}$$

and the two eigenvalue sets are related by

$$\mu_{kn}^2 = \nu_k^2 + \lambda_n^2 \tag{21}$$

From equation (10)

$$\frac{1}{C_n} = \int_0^1 \phi_n^2(r)rdr = \frac{\lambda_n^2 + h_3^2}{2\lambda_n^2} \tag{22}$$

and

$$\frac{1}{L_k} = \int_{-\eta}^{\eta} \chi_k^2(z)dz = \frac{\eta\left(\nu_k^2+h_1^2\right)\left(\nu_k^2+h_2^2\right) + \frac{1}{2}\left(h_1+h_2\right)\left(\nu_k^2+h_1h_2\right)}{\nu_k^2\left(\nu_k^2+h_2^2\right)} \qquad \text{for } \nu_k \neq 0$$

$$= 2\eta \qquad \text{for } \nu_k = 0 \tag{23}$$

The temperature expression of equation (14) and its attendant definitions (equations (17), (18), (19), (20), (22), and (23)) are still much more general than necessary to solve the laser heating problem. A few observations will reduce equation (14) to a much more manageable size. First, in the typical laser experiment there is initially no temperature profile in the body being irradiated, so that integral containing the function F(r,z) can be dropped.

Second, by assuming that the material is a good absorber of laser energy, we are irradiating only the front surface of the cylinder $z = \eta$ with flux $f_2(r,\tau)$, and therefore, $f_1(r,\tau)$ and $f_3(z,\tau)$ are both zero. The integral containing $Q(r,z,\tau)$ also vanishes because no heat is being generated in the bulk.* Third, by assuming that the coefficients of combined reradiation and convection are the same for all cylinder surfaces $h_1=h_2=h_3=h$, equations (20) and (23) are simplified slightly. We now rewrite equation (14) with its accompanying definitions for the laser heating problem.

$$T(r,z,t) = \sum_k^\infty \sum_n^\infty C_n L_k \phi_n(r) \chi_k(z) \exp\left[-\left(\lambda_n^2+\nu_k^2\right) t\right]$$

$$\cdot \left\{ \int_0^1 \int_0^t \exp\left[\left(\lambda_n^2+\nu_k^2\right)\tau\right] \phi_n(r)\, \chi_k(\eta) f_2(r,\tau) r\,dr\,d\tau \right\} \tag{24}$$

where

$$\phi_n(r) = J_0\left(\lambda_n r\right)/J_0\left(\lambda_n\right) \tag{25}$$

λ_n is the nth non-negative root of

$$hJ_0\left(\lambda_n\right) = \lambda_n\, J_1\left(\lambda_n\right) \tag{26}$$

$$\chi_k(z) = \cos\left[\nu_k(\eta+z)\right] + \left(h/\nu_k\right)\sin\left[\nu_k(\eta+z)\right] \tag{27}$$

ν_k is the kth non-negative root of

$$2h\nu_k \cos 2\eta\nu_k = \left(\nu_k^2-h^2\right)\sin 2\eta\nu_k \tag{28}$$

---

*For dielectric materials with very long absorption lengths at the laser wavelenth of interest, the absorption of energy in the bulk could be handled by retaining the heat generation term $Q(r,z,\tau)$ and setting the three surface fluxes $f_1(r,\tau)$, $f_2(r,\tau)$, and $f_3(z,\tau)$ to zero.

$$C_n = 2\lambda_n^2/\left(\lambda_n^2+h^2\right) \tag{29}$$

$$L_k = \nu_k^2/\left[\eta\left(\nu_k^2+h^2\right)+ h\right] \qquad \nu_k \neq 0$$

$$= 1/2\eta \qquad \nu_k = 0 \tag{30}$$

The temperature expression summarized by equations (24) through (30) is still a very general solution for the heating of a cylinder by a laser, because the only requirement made is that the flux entering the front surface be radially symmetric. The flux, represented by $f_2(r,\tau)$, is still a general function of radius and time. It is desirable, of course, to choose beam intensity profiles in time and space which when substituted into equation (24) permit a direct analytical evaluation of the double integral in the braces { } because this integration is inside of the double sum and must be performed n•k times.

2. LASER BEAM SPACIAL INTENSITY PROFILES

The spacial integration of equation (24) has been evaluated for three laser beam profiles: constant intensity, f_0', of finite radius (cylindrical profile), Gaussian, and parabolic. For the cylindrical profile,

$$f_2'(r) = f_0' \qquad 0 \leq r' \leq \sigma'$$

$$f_2'(r) = 0 \qquad \sigma' < r'$$

and the total beam power, P', is

$$P' = \pi\sigma'^2 f_0' \tag{31}$$

Normalizing the appropriate parameters the integral to be performed is

$$\int_0^\sigma \frac{J_0\left(\lambda_n r\right)}{J_0\left(\lambda_n\right)} f_0 \, r dr = \frac{f_0 \sigma \, J_1\left(\lambda_n \sigma\right)}{\lambda_n \, J_0\left(\lambda_n\right)} \tag{32}$$

For the Gaussian profile

$$f_2'(r) = f_0' \exp(-r'^2/2\sigma'^2) \qquad 0 \leq r' \leq a'$$

and

$$P' = 2\pi\sigma'^2 f_0' \tag{33}$$

With normalized parameters the required integral is

$$\int_0^1 \frac{J_0(\lambda_n r)}{J_0(\lambda_n)} f_0 \exp(-r^2/2\sigma^2) r dr$$

This integration cannot be completed in closed form; however, an exact expression is available for the definite integral from zero to infinity.

$$\int_0^\infty J_\nu(\beta r) r^{\nu+1} \exp(-\gamma^2 r^2) dr = \frac{\beta^\nu}{(2\gamma^2)^{\nu+1}} \exp(-\beta^2/4\gamma^2) \tag{34}$$

As long as the effective radius of the beam is somewhat smaller than the sample radius, the exponential part of the function to be integrated has decayed to negligible values for $r = 1$, and thus, integrating to 1 is equivalent to integrating to ∞. Applying equation (34) for $\nu = 0$,

$$\int_0^1 \frac{J_0(\lambda_n r)}{J_0(\lambda_n)} f_0 \exp(-r^2/2\sigma^2) r dr \simeq \frac{f_0 \sigma^2}{J_0(\lambda_n)} \exp\left(-\sigma^2 \lambda_n^2/2\right) \tag{35}$$

For the parabolic profile

$$f_2'(r) = f_0'(1 - r'^2/\sigma'^2) \qquad 0 \leq r' \leq \sigma'$$

$$f_2'(r) = 0 \qquad \sigma' < r'$$

and

$$P' = \pi\sigma'^2 f_0'/2 \tag{36}$$

The required integral is evaluated with an integration by parts

$$\int_0^{\sigma} \frac{J_0\left(\lambda_n r\right)}{J_0\left(\lambda_n\right)} f_0(1 - r^2/\sigma^2)rdr = \frac{2f_0\ J_2\left(\lambda_n \sigma\right)}{\lambda_n^2\ J_0\left(\lambda_n\right)} \tag{37}$$

3. LASER BEAM TEMPORAL INTENSITY PROFILES

The temporal integration of equation (24) has been performed for three different kinds of lasers, continuous wave (CW), pulsed, and repetitively pulsed (REP). The REP solution works for a very general pulse shape as each pulse is represented by a rising and a decaying exponential. Two types of single pulse solutions are easily derived. The first, a simple square pulse is a specialization of the CW solution, while the second is obtained directly from the REP solution by setting the pulse frequency to a number sufficiently small to allow only one pulse of energy during the times of interest.

For the CW laser beam

$$f_2'(t') = f_0 \qquad 0 \leq t'$$

and the time integration in equation (24) is just

$$\int_0^t \exp\left[\left(\lambda_n^2+\nu_k^2\right)\tau\right] f_0 d\tau = \frac{f_0}{\lambda_n^2+\nu_k^2}\left\{\exp\left[\left(\lambda_n^2+\nu_k^2\right)t\right] - 1\right\} \tag{38}$$

The single square pulse beam

$$f_2'(t') = f_0 \qquad 0 \leq t' \leq t_p'$$

$$f_2'(t') = 0 \qquad t_p' < t'$$

requires only a change of the upper limit of equation (38) to, t_p, the normalized pulse length. (38a)

A temperature solution is possible for the REP laser if it is assumed that the pulses are all of identical energy, shape, and spacing in time. The following general shape was chosen for a pulse.

$$\text{rise:} \quad f_2'(t') = f_0' \left[1-\exp\left(-t'/t'_{pr}\right)\right] \qquad 0 < t' \leq t'_{pp}$$

$$\text{decay:} \quad f_2'(t') = f_0' \exp\left[-\left(t'-t'_{pp}\right)/t'_{pd}\right] \qquad t'_{pp} < t'$$

Three time constants are required to describe each pulse: the rise time, t'_{pr}, the time, t'_{pp}, for the intensity to reach a maximum, f_0', and the decay time, t'_{pd}. The energy per unit area contained in this pulse is given by

$$\frac{\text{Energy}}{\text{Unit area}} = f_0'\left[t'_{pp}-t'_{pr}+t'_{pd}+t'_{pr}\exp\left(-t'_{pp}/t'_{pr}\right)\right] \tag{39}$$

Thus, for the mth pulse in a series of pulses separated by a period, θ',

$$\text{rise:} \quad f_2'(t') = f_0'\left[1-\exp\left(-\left(t'-(m-1)\theta'\right)/t'_{pr}\right)\right]$$

$$(m-1)\theta' < t' \leq (m-1)\theta' + t'_{pp}$$

$$\text{decay:} \quad f_2'(t') = f_0' \exp\left[-\left(t'-(m-1)\theta'-t'_{pp}\right)/t'_{pd}\right]$$

$$(m-1)\theta' + t'_{pp} < t' \leq m\theta'$$

The time integral of equation (24) where t' is a time during the Mth pulse

$$\int_0^t \exp\left[\left(\lambda_n^2+\nu_k^2\right)\tau\right] f_2(\tau)d\tau$$

therefore, consists of first, a complete integration over pulses 1 through M-1, and second, integration over the Mth pulse up to time t'. Substituting a normalized form of f_2' (t') for pulses 1 through M-1 and for pulse M into the time integral and letting the sum of the squares of the eigenvlaues, $\lambda_n^2+\nu_k^2$, be represented by E_{nk} one obtains

$$\int_0^t \exp\left(E_{nk}\tau\right) f_2(\tau)\, d\tau = \sum_{m=1}^{M-1} \exp\left[E_{nk}\left((m-1)\theta - t\right)\right] \left\{ \frac{f_0}{E_{nk}} \left[\exp\left(E_{nk} t_{pp}\right) - 1\right] \right.$$

$$- \frac{f_0 t_{pr}}{E_{nk} t_{pr} - 1} \left[\exp\left[\left(E_{nk} - 1/t_{pr}\right) t_{pp}\right] - 1\right]$$

$$+ \frac{f_0 t_{pd}}{E_{nk} t_{pd} - 1} \exp\left(t_{pp}/t_{pd}\right) \left[\exp\left[\left(E_{nk} - 1/t_{pd}\right)\theta\right]\right.$$

$$\left.\left. - \exp\left[\left(E_{nk} - 1/t_{pd}\right) t_{pp}\right]\right]\right\} \qquad 0 < t \leq (M-1)\theta$$

either

$$+ \frac{f_0}{E_{nk}} \left[1 - \exp\left[E_{nk}\left((M-1)\theta - t\right)\right]\right]$$

$$- \frac{f_0 t_{pr}}{E_{nk} t_{pr} - 1} \left[\exp\left[\left((M-1)\theta - t\right)/t_{pr}\right] - \exp\left[E_{nk}\left((M-1)\theta - t\right)\right]\right]$$

$$(M-1)\theta < t \leq (M-1)\theta + t_{pp}$$

(a time during the rise of the Mth pulse)

or

$$+ \exp\left[E_{nk}\left((M-1)\theta - t\right)\right] \left\{ \frac{f_0}{E_{nk}} \left[\exp\left(E_{nk} t_{pp}\right) - 1\right] \right.$$

$$\left. - \frac{f_0 t_{pr}}{E_{nk} t_{pr} - 1} \left[\exp\left[\left(E_{nk} - 1/t_{pr}\right) t_{pp}\right] - 1\right]\right\}$$

$$+ \frac{f_0 t_{pd}}{E_{nk} t_{pd} - 1} \left[\exp\left[\left((M-1)\theta + t_{pp} - t\right)/t_{pd}\right] - \exp\left[E_{nk}\left((M-1)\theta + t_{pp} - t\right)\right]\right]$$

$$(M-1)\theta + t_{pp} < t < M\theta$$

(a time during the decay of the Mth pulse)

(40)

The first part of equation (40) containing $\sum_m$ is easier to evaluate than it appears at first because the index, m, and the variable, t, are contained only in the first exponential term, and the argument is always negative. Since REP lasers to date operate in the order of hundreds of pulses per second or less, M, the total number of pulses is never an unreasonably large number. Furthermore the expression contained in the braces is just a double indexed set of constants to be evaluated once after the eigenvalues are found. The choice of the appropriate second part of equation (40) is made directly from the variable t.

As stated earlier, this solution can be used in the single pulse case by entering a period, θ, which is always greater than t. Only the appropriate second part of equation (40) is to be evaluated for M=1.

SECTION III

THE PROGRAM

To obtain a solution to equation (24) one must consider sufficiently many terms of the series to allow for convergence. In general, the number of terms required for convergence is large enough to make any solution other than by a high-speed computer impractical. The program which is described below and appears in the appendix was written to provide an evaluation of equation (24). This program was written in FORTRAN IV source language and is run on a CDC 6600. The following is a list of symbols used in the program.

CAP	κ
CØND	K
A	a
D	2b
SIG	σ'
FL	f'_0
H	h'
ELAM	λ
ENU	ν
P	P'
TPR	t'_{pr}
TPP	t'_{pp}
TPD	t'_{pd}
EGN	$\lambda_m^2 + \nu_k^2 = E_{nk}$

The main program, called TMPC, initializes input data through the use of data statements. The desired times of interest are generated prior to any call to subroutines. In addition, one takes into consideration the relationship

between peak intensity and power as described in equations (31), (33), and (36) according to the particular spacial and temporal beam profile of interest.*

After these preliminary values have been determined, then a call is made to INITL. This subroutine first computes the heat transfer coefficient H, and then normalizes various constants. Finally, two calls to the subroutine ZERØ are made: one for the function TRANT, and one for the function TRANB.

Subroutine ZERØ determines a preassigned number of roots to a desired accuracy for a given function. The function TRANT evaluates equation (28) for the particular value of ν which is passed to it. Similarily, TRANB evaluates equation (26) for different values of λ. The number of ν_k's and λ_n's calculated is determined by NEGT and NEGB, respectively. It has been found that 2000 ν_k's and 300 λ_n's will normally produce a temperature accuracy within 3×10^{-4} degrees centigrade with the time required to calculate them, usually in the range of a fraction of a second. If computer time becomes excessive these numbers, in many cases, can be substantially decreased. The number of terms actually needed is determined primarily by the magnitude of the temperature gradient at the particular time of interest. For example, the number of λ_n's required for the cylindrical beam at short times is significantly greater than the number required in the Gaussian case at the same times. After the ν_k's and the λ_n's have been calculated control is returned to the main program.

A call is then made to the function routine TMP which will compute the temperature, as given in equation (24), for a particular radius, depth, and time. In TMP a desired accuracy is set for the temperature calculation, usually 3×10^{-4}, and the radius, depth, and time are normalized. The function then begins to evaluate the first term of the double sum of equation (24) by setting the indices k and n to 1. ARGA and ARGB are then calculated to be later used as the arguments in the computation of ϕ_n as defined in equation (25). Next a call is made to function routine SINT which does the spacial integration of equation (24) for either a cylindrical profile (equation (32)), Gaussian profile (equation (35)), or parabolic profile (equation (37)). The program next computes the k,nth term of the double sum in the following order.

*There are two sample programs listed in the appendix: one for a CW laser with a cylindrical beam profile, and another for a REP laser with a Gaussian beam profile.

First, the functional constant C_n, evaluated in function routine C, is determined as defined in equation (29). The next term is the functional constant L_k, evaluated in function routine SL, as defined in equation (30). The previously determined arguments, ARGA and ARGB, are used in the computation of the eigenfunction, ϕ_n. The following two terms of the product are χ_k functions found in equation (26) and evaluated in the function routine CHI.

The product of the temporal integration and the time dependent exponential term of equation (24) computed in function routine TFACT, comprises the next term of the product. The temporal integration will be given by either equation (38), (38a), or (40) according to whether the laser beam is CW, single pulse, or multiple pulse. Here "a" denotes a change in the upper limit of equation (38) to t_p. The final factor is the result of the spacial integration previously computed. The program will continue to run through this double sum until either the specified accuracy is achieved or the number of eigenvalues requested is exceeded. Once one of those conditions has been met, in the latter case an error message will be printed, then control is returned to the main program. Repeated calls to TMP will produce the desired temperatures and all that remains is the output of this information.

A typical output from this program might include thermal properties of the substance under consideration, dimensions of the sample, beam parameters, and the respective times and temperatures at specified locations.

The program usually will compute 10 temperatures on a CDC 6600 in about 1 or 2 seconds.

SECTION IV
APPLICATIONS

1. DIRECT CALCULATION OF TEMPERATURES

Direct calculation of temperature time histories at any number of locations or temperature profiles at any times are possible with a minimum expenditure of computer time. Because this is a series analytical solution it is necessary to calculate only the temperatures at the locations and times desired, rather than generate the whole temperature field at every time step up to the time of interest as done in finite element or finite difference computer solutions. This analytical solution is particularly useful for calculating heat flow in a variety of laser effects experiments. Temperature rises can be estimated for two-dimensional CW, pulsed, and REP lasers. The effect of temporal pulse shape on the temperature history can be investigated in detail, and in the case of REP lasers, the amount of radial and axial "recovery cooling" of the irradiated area occurring between pulses can also be calculated.

In addition to the investigation of various time parameters characterizing the laser flux, the importance of the laser beam's spacial profile can also be studied. For a laser beam of a given total power, the beam size parameter, σ, and peak intensity, f_0, can be varied to determine the effects of focusing or defocusing the laser beam on the temperature profiles in a sample. Also temperature fields created by the different beam spacial profiles can be compared, at least for the cylindrical, Gaussian, and parabolic profiles derived here. Certainly other mathematical functions can be chosen to represent the beam profile of interest. With several possible functions to try, one can compare the resulting temperature field and determine quantitatively how important is the precise representation of the laser beam profile to the predicted form of the temperature field.

2. DETERMINATION OF SURFACE ABSORPTION COEFFICIENTS

In the study of the effects of laser beams on materials it is nearly always desirable to know what fraction of the incoming laser beam is being absorbed by the sample. This surface absorption coefficient can be obtained from experimental temperature time histories with the aid of this heat transfer solution.

For example, temperature data from thermocouples attached to a laser irradiated sample can be fit by a method of "least squares" with the mathematical form of the temperature rise predicted by the two-dimensional program. Knowing the actual beam power, the program can output what fraction of that power the sample must have absorbed.

This data fitting technique permits one to obtain surface absorption coefficients for the wide range of laser effects experiments where the assumption of one-dimensional heat flow cannot be made. In fact the amount of radial heat losses in most laser experiments is considerable, and if not accounted for properly can result in enormous errors in experimental absorption coefficients.

3. OTHER APPLICATIONS

Because of the remarkably general nature of this two-dimensional heat transfer solution as expressed in equation (14), one can envision a variety of other applications. Rather than assuming surface heating, the laser energy could be absorbed over finite depths in the sample through the use of the heat generation integral. More sophisticated solutions might then be developed which would model the heating of materials semitransparent to laser radiation such as lenses, windows, etc.

APPENDIX I

CONTINUOUS WAVE LASER,
CYLINDRICAL BEAM PROFILE

```
      PROGRAM TMPC(INPUT,OUTPUT)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),FLAM(1000),FNU(2000)
      COMMON /DATAS/ DATX(100),DATY(100),NPTS,X
      COMMON /NUSES/ NNU,NLA,TPD
      DATA CAP,COND,A,D,SIG/0.52,1.21,5.00,0.0508,0.95/
      DATA NEGT,NEGB/2000,300/
      DATA ABC,TREF/0.1,300./
      DATA PI/3.141592653589793/
      DIDENT=9H2024 ALUM
      NPTS=51
      DLT=0.1
      DO 1 I=1,NPTS
      DATX(I)=DLT*(I-1)
1     CONTINUE
      P=2000.
      FL=P/(1.*PI*SIG*SIG)
      X=0.0
      FLD=FL*A/(TREF*COND)
      ALPHA=0.025
      FLD=FLD*ALPHA
      CALL INITL
      DO 2 I=1,NPTS
      F(I)=TMP(X,D,DATX(I))
2     CONTINUE
      PRINT 3, DIDENT,CAP,COND,A,D,P,FL,SIG
      PRINT 4, ALPHA
      PRINT 5, (DATX(I),F(I),I=1,NPTS)
C
3     FORMAT (1H1,A9///,40H THERMAL DIFF(CM2/SEC). . . . . . . . . ,F10.
     14/,40H THERMAL COND(JOULES/(CM)(SEC)(DEG C)). ,1F10.4/,40H TARGET
     2RADIUS(CM). . . . . . . . . . . . ,F10.4/,40H THICKNESS(CM). . . . .
     3 . . . . . . . . . ,F10.4/,40H POWER(WATTS) . . . . . . . . . . . . .
     4. ,F10.4/,40H PEAK INTS(WATTS/CM2) . . . . . . . . . ,F10.4/,40H S
     5IGMA(CM). . . . . . . . . . . . . . . ,F10.4)
4     FORMAT (///,23HWITH ABSORPTION COEFF =,F10.4,///)
5     FORMAT (6H TIME=,F10.4,5X,5HTEMP=,F10.4,/)
      END
```

```
      SUBROUTINE INITL (P)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,RFSB,F(500),ELAM(1000),ENU(2000)
      EXTERNAL TRANT,TRANB
      H=(4.*(TREF**3)*ABC*5.669E-12)*A/COND
      ETA=D/(2.*A)
      SIGD=SIG/A
      DIV=1./ETA
      DIVA=SQRT(H/ETA)/3.
      DIVB=SQRT(H/3.)
      CALL ZERO (TRANT,NEGT,ENU,1.0E-10,DIV,DIVA)
      CALL ZERO (TRANB,NEGB,ELAM,1.0E-10,1.,DIVB)
      RETURN
      END
      SUBROUTINE ZERO (F,N,ARRAY,ACCUR,DIV,DIVA)
C     ZEROS IN ON EIGENVALUES LAMBDA(N) AND NU(K)
      DIMENSION ARRAY(N)
      STEP=DIVA
      ISTART=1
      X=STEP
      THIS=F(X)
      DO 3 I=ISTART,N
1     PREV=THIS
      X=X+STEP
      THIS=F(X)
      IF (SIGN(1.,THIS).EQ.SIGN(1.,PREV)) GO TO 1
      ZSTEP=-STEP/2.
2     X=X+ZSTEP
      THIS=F(X)
      IF (ABS(THIS).GE.ABS(PREV).AND.ABS(ZSTEP).LE.(STEP/10.)) GO TO 1
      ZSTEP=ABS(ZSTEP)/2.
      IF (SIGN(1.,THIS).NE.SIGN(1.,PREV)) ZSTEP=-ZSTEP
      IF (ABS(THIS).GE.ACCUR) GO TO 2
      ARRAY(I)=X
      IF (I.GE.1) STEP=DIV
      X=X+STEP
      THIS=F(X)
3     CONTINUE
      RETURN
      END
```

```
      FUNCTION TMP (RP,ZP,TP)
C     COMPUTES TEMPERATURE
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      COMMON /NUSES/ NNU,NLA,TPD
      DATA ACCUR/3.0E-4/
      R=RP/A
      Z=(D/2.-ZP)/A
      T1=TP
      T=TP*CAP/(A**2)
      SUMA=0.
      SUM=0.
      SCA=0.
      SCD=0.
      K=1
      N=1
    1 CONTINUE
      ARGA=ELAM(N)*R
      ARGB=ELAM(N)
      ANS=SINT(N)
    2 CONTINUE
      EGN=ELAM(N)**2+ENU(K)**2
      ADDN=C(N)*SL(K)*(BF(ARGA)/BF(ARGB))*CHI(K,Z)*CHI(K,ETA)*TFACT(T,EG
     1N)*ANS
      SUM=SUM+ADDN
      SCD=SCD+ADDN
      IF (ABS(SCD).LE.ABS(SUM*ACCUR)) GO TO 3
      IF (K.EQ.NEGT) GO TO 3
      K=K+1
      SCD=ADDN
      GO TO 2
    3 CONTINUE
      SUMA=SUMA+SUM
      SCA=SCA+SUM
      IF (ABS(SCA).LE.ABS(SUMA*ACCUR/30)) GO TO 5
      IF (N.EQ.NEGB) GO TO 4
      N=N+1
      K=1
      SCA=SUM
      SCD=0.
```

```
      SUM=0.
      GO TO 1
4     CONTINUE
      PRINT 7, T1
5     CONTINUE
      IF (K.EQ.NEGT) PRINT 6, T1
      TMP=SUMA*TREF
      IF (K.GT.NNU) NNU=K
      IF (N.GT.NLA) NLA=N
      RETURN
C
6     FORMAT (32H MAX NO OF NU S USED FOR T(SEC)=,F16.8,27H     TEMP MAY
     1 BE INACCURATE)
7     FORMAT (33H MAX NO LAMBDA S USED FOR T(SEC)=,F16.8,28H      TEMP M
     1AY BE INACCURATE)
      END
      FUNCTION TFACT (T,EGN)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      TFACT=FLD*(1.-EXP(-EGN*T))/EGN
      RETURN
      END
      FUNCTION SINT (M)
C     SQUARE BEAM PROFILE
C     COMPUTES INTEGRAL OF PHI(N) AND FLUX(RADIUS,TIME)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      IF (SIG.GE.A) GO TO 1
      SINT=SIGD*BI(SIGD*ELAM(M))/(ELAM(M)*BF(ELAM(M)))
      RETURN
1     CONTINUE
      SINT=BI(ELAM(M))/(ELAM(M)*BF(ELAM(M)))
      RETURN
      END
      FUNCTION TRANT (X)
C     COMPUTES VALUES FOR NU(K)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      TRANT=TAN(2.*X*ETA)-2.*H*X/(X**2-H**2)
      RETURN
      END
```

```
      FUNCTION TRANB (X)
C     COMPUTES VALUES FOR LAMBDA(N)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      TRANB=X*R1(X)-H*BF(X)
      RETURN
      END
      FUNCTION CHI (M,PT)
C     CHI FUNCTION
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      ARG=(ETA+PT)*ENU(M)
      IF (ARG.EQ.0.) GO TO 1
      CHI=COS(ARG)+(H/ENU(M))*SIN(ARG)
      RETURN
1     CHI=1.
      RETURN
      END
      FUNCTION SL (M)
C     COMPUTES FUNCTIONAL CONSTANT L(K)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      IF (ENU(M).EQ.0.) GO TO 1
      SL=(ENU(M)**2)/(ETA*(ENU(M)**2+H**2)+H)
      RETURN
1     SL=1./(2.*ETA)
      RETURN
      END
      FUNCTION C (M)
C     COMPUTES FUNCTIONAL CONSTANT C(N)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      C=(2.*ELAM(M)**2)/(ELAM(M)**2+H**2)
      RETURN
      END
```

```
      FUNCTION B1 (A)                                                   L   1
C     BESSEL FUNCTION*****KIND 1 AND  ORDER 1                           L   2
      X=A/3.                                                            L   3
      IF (A.GT.3.) GO TO 1                                              L   4
      Y=X*X                                                             L   5
      B1=A*(.5+Y*(-.56249985+Y*(.2109357+Y*(-.03954289+Y*(.00443319+Y*(-L   6
     1.00031761+Y*(.00001109)))))))                                     L   7
      RETURN                                                            L   8
1     CONTINUE                                                          L   9
      Y=1./X                                                            L  10
      F=.79788456+Y*(.00000156+Y*(.01659667+Y*(.00017105+Y*(-.00249511+Y L  11
     1*(.00113653+Y*(-.00020033))))))                                   L  12
      Y=1./X                                                            L  13
      TH=A-2.35619449+Y*(.12499612+Y*(.0000565+Y*(-.00637879+Y*(.0007434 L  14
     18+Y*(.00079824+Y*(-.00029166))))))                                L  15
      B1=F*COS(TH)/SQRT(A)                                              L  16
      RETURN                                                            L  17
      END                                                                  /
      FUNCTION BF (A)                                                   M   1
C     BESSEL FUNCTION*****KIND 1 AND  ORDER 0                           M   2
      X=A/3.                                                            M   3
      IF (A.GT.3.) GO TO 1                                              M   4
      Y=-X*X                                                            M   5
      BF=1.+Y*(2.2499997+Y*(1.2656208+Y*(0.3163866+Y*(0.0444479+Y*(.0039 M   6
     1444+Y*(.00021))))))                                               M   7
      RETURN                                                            M   8
1     CONTINUE                                                          M   9
      Y=1./X                                                            M  10
      F=.79788456+Y*(-.00000077+Y*(-.0055274+Y*(-.00009512+Y*(.000137237 M  11
     1+Y*(-.00072805+Y*(.00014476))))))                                 M  12
      TH=A-.78539816+Y*(-.04166397+Y*(.00003954+Y*(.0026257+Y*(-.0005412 M  13
     15+Y*(-.00029333+Y*(.00013558))))))                                M  14
      BF=F*COS(TH)/SQRT(A)                                              M  15
      RETURN                                                            M  16
      END                                                                  /
```

APPENDIX II

REPETITIVELY PULSED LASER, GAUSSIAN BEAM PROFILE

```
      PROGRAM MPULSE(INPUT,OUTPUT)
      COMMON /SPEC/ A,D,ARC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      COMMON /NUSES/ NNU,NLA,TPD
      DIMENSION DATX500)
      DATA CAP,COND,A,D,SIG/0.52,1.21,2.54,0.0406,0.275/
      DATA NEGT,NEGB/2000,350/
      COMMON /DATAS/ DATX(200),DATY(200),NPTS,X
      DATA ARC,TREF/0.1,300./
      DATA PI/3.141592653589793/
      CALL INITL (P)
      SIGD=SIG/A
      TPD=1.0E-6
      TPP=30.6E-6$TPR=0.50E-6$TH=100.0
      ENG=2.1
      FL=ENG/(TPP-TPR+TPD+TPR*EXP(-TPP/TPR))*1*PI*SIG**2
      FLD=FL*A/(TREF*COND)
      ALPHA=0.10
      FLD=FLD*ALPHA
      TPD=TPD*CAP/A**2
      PRINT 3, FL,ENG
      DLT=1.0E-2
      S=X=0.0
      DO 1 I=1,11
      DATX(I)=(I-1)*DLT
    1 CONTINUE
      PRINT 5, S
      PRINT 6
      DO 2 I=1,11
      F(I)=TMP(X,S,DATX(I))
      PRINT 7, (DATX(I),F(I))
    2 CONTINUE
      PRINT 4, NNU,NLA
      STOP
C
    3 FORMAT (1H1,11H PEAK INST=,F12.4/,8H ENERGY=,F12.4)
    4 FORMAT (13H NO NU S USED,I5,5X,15HNO LAMDA S USED,I5)
    5 FORMAT (7H DEPTH=,F8.4)
    6 FORMAT (5X,4HTIME,15X,4HTEMP)
    7 FORMAT (5X,F16.8,5X,F16.8)
      END
```

```
      SUBROUTINE INITL (P)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),ELAM(1000),ENU(2000)
      EXTERNAL TRANT,TRANB
      H=(4.*(TREF**3)*ABC*5.669E-12)*A/COND
      ETA=D/(2.*A)
      DIV=1./ETA
      DIVA=SQRT(H/ETA)/3.
      DIVB=SQRT(H/3.)
      CALL ZERO (TRANT,NEGT,ENU,1.0E-08,DIV,DIVA)
      CALL ZERO (TRANB,NEGB,ELAM,1.0E-08,1.,DIVB)
      RETURN
      END
      SUBROUTINE ZERO (F,N,ARRAY,ACCUR,DIV,DIVA)
C     ZEROS IN ON EIGENVALUES LAMBDA(N) AND NU(K)
      DIMENSION ARRAY(N)
      STEP=DIVA
      ISTART=1
      X=STEP
      THIS=F(X)
      DO 3 I=ISTART,N
    1 PREV=THIS
      X=X+STEP
      THIS=F(X)
      IF (SIGN(1.,THIS).EQ.SIGN(1.,PREV)) GO TO 1
      ZSTEP=-STEP/2.
    2 X=X+ZSTEP
      THIS=F(X)
      IF (ABS(THIS).GE.ABS(PREV).AND.ABS(ZSTEP).LE.(STEP/10.)) GO TO 1
      ZSTEP=ABS(ZSTEP)/2.
      IF (SIGN(1.,THIS).NE.SIGN(1.,PREV)) ZSTEP=-ZSTEP
      IF (ABS(THIS).GE.ACCUR) GO TO 2
      ARRAY(I)=X
      IF (I.GE.1) STEP=DIV
      X=X+STEP
      THIS=F(X)
    3 CONTINUE
      RETURN
      END
```

```
      FUNCTION TMP (RP,ZP,TP)
C     COMPUTES TEMPERATURE
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),ELAM(1000),ENU(2000)
      COMMON /NUSES/ NNU,NLA,TPD
      DATA ACCUR/3.0E-4/
      R=RP/A
      Z=(D/2.-ZP)/A
      T1=TP
      T=TP*CAP/(A**2)
      SUMA=0.
      SUM=0.
      SCA=0.
      SCD=0.
      K=1
      N=1
    1 CONTINUE
      ARGA=ELAM(N)*R
      ARGR=ELAM(N)
      ANS=SINT(N)
    2 CONTINUE
      EGN=ELAM(N)**2+ENU(K)**2
      ADDN=C(N)*SL(K)*(BF(ARGA)/BF(ARGR))*CHI(K,Z)*CHI(K,ETA)*TFACT(T,EG
     1N)*ANS
      SUM=SUM+ADDN
      SCD=SCD+ADDN
      IF (ABS(SCD).LE.ABS(SUM*ACCUR)) GO TO 3
      IF (K.EQ.NEGT) GO TO 3
      K=K+1
      SCD=ADDN
      GO TO 2
    3 CONTINUE
      SUMA=SUMA+SUM
      SCA=SCA+SUM
      IF (ABS(SCA).LE.ABS(SUMA*ACCUR/30)) GO TO 5
      IF (N.EQ.NEGR) GO TO 4
      N=N+1
      K=1
      SCA=SUM
      SCD=0.
```

```
      SUM=0.
      GO TO 1
4     CONTINUE
      PRINT 7, T1
5     CONTINUE
      IF (K.EQ.NEGT) PRINT 6, T1
      TMP=SUMA*TREF
      IF (K.GT.NNU) NNU=K
      IF (N.GT.NLA) NLA=N
      RETURN
C
6     FORMAT (32H MAX NO OF NU S USED FOR T(SEC)=,F16.8,27H     TEMP MAY
     1 BE INACCURATE)
7     FORMAT (33H MAX NO LAMBDA S USED FOR T(SEC)=,F16.8,28H      TEMP M
     1AY BE INACCURATE)
      END
      FUNCTION TFACT (T,EGN)
C     MULTIPLE PULSE ROUTINE
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),FLAM(1000),FNU(2000)
      COMMON /NUSFS/ NNU,NLA,TPD
      TH=100.00*CAP/A**2
      TPP=30.6E-6*CAP/A**2
      TPR=0.50E-6*CAP/A**2
      M=T/TH
      IF (T.EQ.M*TH.AND.T.GT.0.0) M=M-1
      IF (M.EQ.0) GO TO 4
      FC1=1/EGN-EXP(-TPP/TPR)/(EGN-1/TPR)-1/(EGN-1/TPD)
      SM1=0.0
      DO 1 I=1,M
      SM1=SM1+EXP(EGN*(I*TH-TH+TPP-T))
1     CONTINUE
      FC1=FC1*SM1
      FC2=1/(EGN-1/TPR)-1/EGN
      SM2=0.0
      DO 2 I=1,M
      SM2=SM2+EXP(EGN*(I*TH-TH-T))
2     CONTINUE
      FC2=FC2*SM2
      FC3=EXP(TPP/TPD)/(EGN-1/TPD)
      SM3=0.0
```

```
      DO 3 I=1,M
      SM3=SM3+EXP(EGN*(I*TH-T)-TH/TPD)
    3 CONTINUF
      FC3=FC3*SM3
      TERM1=FC1+FC2+FC3
    4 IF (T.GT.M*TH+TPP) GO TO 5
      FC1=(1.-EXP(EGN*(M*TH-T)))/EGN
      FC2=EXP(M*TH/TPR-T/TPR)
      FC3=EXP(EGN*M*TH-EGN*T)
      TERM2=FC1-(FC2-FC3)/(EGN-1/TPR)
      GO TO 6
    5 FC1=1/EGN-EXP(-TPP/TPR)/(EGN-1/TPR)
      FC1=FC1*EXP(EGN*(M*TH+TPP-T))
      FC2=(1/(EGN-1/TPR)-1/EGN)
      FC2=FC2*EXP(EGN*M*TH-EGN*T)
      FC3=EXP((M*TH+TPP-T)/TPD)-EXP(EGN*(M*TH+TPP-T))
      TERM2=FC1+FC2+FC3/(EGN-1/TPD)
    6 TFACT=FLD*(TERM1+TERM2)
      RETURN
      END
      FUNCTION SINT (M)
C     GAUSSIAN BEAM PROFILE
C     COMPUTES INTEGRAL OF PHI(N) AND FLUX(RADIUS,TIME)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      IF (SIG.GE.A) GO TO 1
      SINT=SIGD**2*EXP(-ELAM(M)**2*SIGD**2/2.)/BF(ELAM(M))
      RETURN
    1 CONTINUE
      SINT=B1(ELAM(M))/(ELAM(M)*BF(ELAM(M)))
      RETURN
      END
      FUNCTION TRANT (X)
C     COMPUTES VALUES FOR NU(K)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BESB,F(500),ELAM(1000),ENU(2000)
      TRANT=TAN(2.*X*ETA)-2.*H*X/(X**2-H**2)
      RETURN
      END
```

```
      FUNCTION TRANB (X)
C     COMPUTES VALUES FOR LAMBDA(N)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),FLAM(1000),FNU(2000)
      TRANB=X*B1(X)-H*BF(X)
      RETURN
      END
      FUNCTION CHI (M,PT)
C     CHI FUNCTION
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),ELAM(1000),ENU(2000)
      ARG=(ETA+PT)*ENU(M)
      IF (ARG.EQ.0.) GO TO 1
      CHI=COS(ARG)+(H/ENU(M))*SIN(ARG)
      RETURN
    1 CHI=1.
      RETURN
      END
      FUNCTION SL (M)
C     COMPUTES FUNCTIONAL CONSTANT L(K)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),ELAM(1000),ENU(2000)
      IF (FNU(M).EQ.0.) GO TO 1
      SL=(FNU(M)**2)/(ETA*(ENU(M)**2+H**2)+H)
      RETURN
    1 SL=1./(2.*ETA)
      RETURN
      END
      FUNCTION C (M)
C     COMPUTES FUNCTIONAL CONSTANT C(N)
      COMMON /SPEC/ A,D,ABC,CAP,SIG,SIGD,COND,TREF,NEGT,NEGB,FL,FLD,ETA,
     1H,BFSB,F(500),ELAM(1000),ENU(2000)
      C=(2.*ELAM(M)**2)/(ELAM(M)**2+H**2)
      RETURN
      END
```

```
      FUNCTION B1 (A)
C     BESSEL FUNCTION*****KIND 1 AND  ORDER 1
      X=A/3.
      IF (A.GT.3.) GO TO 1
      Y=X*X
      B1=A*(.5+Y*(-.56249985+Y*(.2109357+Y*(-.03954289+Y*(.00443319+Y*(-
     1.00031761+Y*(.00001109)))))))
      RETURN
1     CONTINUE
      Y=1./X
      F=.79788456+Y*(.00000156+Y*(.01659667+Y*(.00017105+Y*(-.00249511+Y
     1*(.00113653+Y*(-.00020033))))))
      Y=1./X
      TH=A-2.35619449+Y*(.12499612+Y*(.0000565+Y*(-.00637879+Y*(.0007434
     18+Y*(.00079824+Y*(-.00029166))))))
      B1=F*COS(TH)/SQRT(A)
      RETURN
      END
      FUNCTION BF (A)
C     BESSEL FUNCTION*****KIND 1 AND  ORDER 0
      X=A/3.
      IF (A.GT.3.) GO TO 1
      Y=-X*X
      BF=1.+Y*(2.2499997+Y*(1.2656208+Y*(0.3163866+Y*(0.0444479+Y*(.0039
     1444+Y*(.00021))))))
      RETURN
1     CONTINUE
      Y=1./X
      F=.79788456+Y*(-.00000077+Y*(-.0055274+Y*(-.00009512+Y*(.000137237
     1+Y*(-.00072805+Y*(.00014476))))))
      TH=A-.78539816+Y*(-.04166397+Y*(.00003954+Y*(.0026257+Y*(-.0005412
     15+Y*(-.00029333+Y*(.00013558))))))
      BF=F*COS(TH)/SQRT(A)
      RETURN
      END
```

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The heating of a solid body by a laser beam has been analyzed using a generalized two-dimensional solution of the heat conduction equation for a finite right circular cylinder. This series-eigenvalue solution includes time-dependent flux boundary conditions and reradiation from all surfaces. Temperature solutions are derived for three spacial laser beam profiles applied to continuous wave, pulsed, and repetitively pulsed laser heating. Computer programs developed to evaluate these temperature solutions are presented.

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